Today: road map to multi-degree of freedom

State space, 1^{st} order form

e^{At}

Goal: understand $M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = \vec{F}(t)$

Look at simple comparison problem: $M\ddot{\vec{x}} + K\vec{x} = \vec{0} \rightarrow$ core

$\hookrightarrow$ normal modes, any solution is a superposition of normal modes

Forcing with no damping

$M\ddot{\vec{x}} + K\vec{x} = \vec{F}(t)$
$\hookrightarrow \vec{F}_0 \sin \omega t + \vec{F}_c$

$\vec{F}_c \rightarrow$ constant solution

$\vec{F} = \vec{0} \rightarrow *$

$\vec{F} = \bar{F}_0 \sin \omega t$

If $\omega \neq \omega_i \rightarrow$ normal mode frequency

Then $\vec{x}(t) = \bar{x} \sin \omega t$

$$\bar{x} = (-\omega^2 M + K)^{-1} \vec{F}_0$$

<u>Yet to deal with:</u> 1.) damping with no forcing
2.) damping with forcing

<u>Damping:</u> $M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = \vec{0}$

might guess: $\vec{x}(t) = \bar{x} e^{-\lambda t} (A \cos \omega t + B \sin \omega t) \rightarrow$ wishful thinking :(

How do we solve?

First order form $\longrightarrow$ define $\vec{v} = \dot{\vec{x}}$ (1)

New equation: $M\dot{\vec{v}} + C\vec{v} + K\vec{x} = \vec{0}$

$$\dot{\vec{v}} = (M^{-1}C\vec{v} + M^{-1}K\vec{x}) \quad (2)$$

(1), (2) are two first order ODE's

$$\begin{bmatrix} \dot{\vec{x}} \\ \dot{\vec{v}} \end{bmatrix} = \overset{2n}{\underset{2n}{\left[\begin{array}{c|c} [0]_{n\times n} & [I]_{n\times n} \\ \hline [-M^{-1}K]_{n\times n} & [-M^{-1}C]_{n\times n} \end{array}\right]}} \begin{bmatrix} \vec{x} \\ \vec{v} \end{bmatrix}$$

$\vec{z} = \dfrac{\vec{x}}{\vec{v}}$ $\qquad \dot{\vec{z}} = A\vec{z}$ $\qquad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \rightarrow$ not symmetric

- has complex e-values + complex e-vectors unless $C = [0]$

To solve $\dot{\vec{z}} = Az$, guess: $\vec{z}(t) = \bar{z}e^{\lambda t}$

$$\frac{d\bar{z}e^{\lambda t}}{dt} = A\bar{z}e^{\lambda t}$$

$\lambda\bar{z}e^{\lambda t} = A\bar{z}e^{\lambda t}$ $\qquad A\bar{z} = \lambda\bar{z}$

[V D] = eig(A)

$V = [\vec{v}_1 | \vec{v}_2 | \vec{v}_3 \ldots]$ $\qquad D = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & \cdots \\ 0 & \lambda_2 & 0 & 0 & \cdots \\ 0 & 0 & \lambda_3 & 0 & \cdots \end{bmatrix}$

$$\vec{z}(t) = c_1\bar{z}_1e^{\lambda_1 t} + c_2\bar{z}_2e^{\lambda_2 t} + \ldots c_n\bar{z}_ne^{\lambda_n t}$$

$\rightarrow \bar{z}_i$ is complex, λ_i is complex

2 ways to deal with this

1.) pick complex C_i to kill imaginary parts

2.) Take real and separately, imaginary part of solution

Let's look at 1st mode

$\lambda_i t = i\omega t$

$$\vec{z}(t) = e^{(\lambda_r t + \lambda_i t)}$$

$\bar{z} \to$ complex

$$= e^{\lambda_r t}(\cos\omega t + i\sin\omega t)\left[\bar{z}_r + i\bar{z}_i\right]$$

$$\mathrm{Re}(\vec{z}(t)) =$$

$$e^{\lambda r t}\left[\cos\omega t\, \bar{z}_r - \sin\omega t\, \bar{z}_i\right]$$

not equal

there is no mode shape that is just decaying

- still don't know if we have enough eigenvectors

Aside: Freshman Calculus

$\dot{x} = ax$

one way to figure it out: Assume solution has taylor series

$$x = c_0 + ct + c_2 t^2 = \dot{x}|_0 + \frac{1}{2}\ddot{x}|_0 + \frac{1}{3!}\dot{x}|_0 + \ldots$$